Primitive models of partitive division: a replication study

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Many researchers have tried to describe number sense in terms of different components, and a consensus is still not reached (Rezat & Rye Ejersbo, 2018). Usually, conceptual understanding of operations is considered as one of the components (McIntosh, Reys & Reys, 1992; Pitta-Pantazi, 2014) of number sense (here considered as *applied/relational* number sense, Sayers & Andrews, 2015). Yang, Hsu and Huang (2004) specify that the knowledge of the relative effect of operations on numbers (how the four basic operations affect their results) contributes to number sense. Exemplifying, they explain that people may show number sense when they "understand that multiplication does not always produce larger numbers and division does not always produce a smaller value" (ibidem, p. 409). According to McIntosh, Reys and Reys (1992), exposing children only to some models of an operation may lead to incorrect generalizations. For instance, drawing only on the repeated addition model for multiplication (or the repeated subtraction for division) may foster the idea that "multiplication always makes things bigger" (and division always makes things smaller) (Hart, 1981). Fischbein, Deri, Nello and Marino (1985, p. 16) claim that their findings show that "the initial didactical models seem to become so deeply rooted in the learner's mind that they continue to exert an unconscious control over mental behaviour even after the learner has acquired formal mathematical notions that are solid and correct". They refer to these profoundly embodied models as *primitive models*. They sustain that repeated addition is the primitive model for multiplication and that there are two primitive models for division: the partitive and quotative (ibidem). In their study, Fischbein and colleagues (ibidem) use this model to predict the effect of changing numbers in simple word problems. They notice that, for many students, the dividend should be greater than the divisor in partitive division. Their results are cited in a big number of studies on this topic (almost 700 citation according to Google Scholar) and often used as a basis for qualitative studies on smaller samples (e.g. Kouba, 1989; Graeber & Tirosh, 1990; Mulligan & Mitchelmore, 1997).

Some of these studies claim to confirm Fischbein and colleagues' findings (Graeber & Tirosh, 1990), while others are in contrast (Mulligan & Mitchelmore, 1997). We wondered what the reason for contrasting findings might be. Analyzing the work by Fischbein and colleagues (1985), we found some methodological weaknesses that might explain the inconsistence with results by other authors. Certainly, this is not the only explanation, but it is a problem to address. Concerning the subjects involved in their research, Fischbein and colleagues (ibidem) describe their sample as made of 628 pupils (228 in grade 5, 202 in grade 7 and 198 in grade 9) coming from 13 different schools in the same city, Pisa in Italy. There is no more information about the socioeconomical background of those schools and if the 13 different schools are equally distributed among the grades. The instrument used is available in literature, both in the original Italian version (Deri, Nello & Marino, 1983) and in English (Fischbein et al, 1985). The rationale behind the choice of the questions is explained, but then any measure of the validity and reliability of the instrument is not given. Furthermore, for reducing the fatigue effect, the total number of 42 items was divided in two forms (21 items each) and each student responded to just one. There is not any evidence of the equivalence of the two forms and so we are not sure if the comparison between items coming from different forms is legitimate.

A replication study: method and instruments

We decided to replicate the study by Fischbein and colleagues (1985) starting from grade 7, proposing some variation at the methodological level and applying statistical methods to test the instruments. We administered Fischbein et al.,'s (1985) test in a computer-based version. This allowed us to involve in our research 22 schools from different part of northern Italy, with different socio-economical background. We had also the possibility to further reduce the order effect: the order of items in our test was randomized for each student, while in Fischbein's work there were only two possible orders for each form. As in the original research, the two forms were distributed randomly to the students, but this does not directly guarantee the comparability of the two groups of students and then of their answers. Hence, we used unvaried items, included in both the two forms of the test, as an anchor allowing to compare the results of the two groups of students. Furthermore, we added four new unvaried items to improve the robustness of the anchor. Finally, we performed little variation in few of the problems to bring them closer to students' nowadays experience and language. We collected data and classified students answers as correct answers, missing answers and type of wrong answers (Inversion of the terms, Wrong numbers, Reverse operation, Other non-equivalent operation). Our sample is composed by 902 grade 7 students (485 responded to the first form and 417 to the second one) belonging to 46 different classes with different backgrounds.

The reliability analysis after the anchoring technique (implemented with the software JMetrik) highlights good internal consistency in terms of Guttman's L2 coefficient (0,84) and acceptable in terms of Cronbach alpha (0,75). Furthermore, large reliability and separation, on both persons and item sides, indicate that the sample is large enough to consistently rank items by difficulty and respondents by ability (Meyer, 2014). Due to pages limitations, in this paper we present only the results of the four items involving partitive division. In each of the two forms of the test, we have one standard partitive division word-problem (e.g. "In 8 boxes there are 96 bottles of mineral water. How many bottles are in each box?") and approximately 90% of the students give the correct answer to these two problems, confirming Fishbein's results. Also, each form included a word-problem with the divisor larger than the dividend (so violating one of the primitive models) (e.g. "15 friends together bought 5 kg of cookies. How much did each one get?").

Results

Approximately 70% of students answer with the right operation, but with an inversion of the order of terms, so reporting an operation which is in line with the primitive model of partitive division. In our study, the percentage drop between the two standard items and those violating the primitive models, is even stronger than in Fischbein's study and the percentage of correct answers decreases to less than 20%. Furthermore, using the anchoring technique, we can directly compare items' difficulty because all the items of the two forms are located on the same scale: the last two items are significantly more difficult than the first two and we notice a change in the difficulty parameter from -2.9/-1.6 of the first two items to +3.2/+3.0 of the second ones (with a standard error around 0.2). We notice also small differences in the percentage of incorrect answers: we have almost the same percentage of inversion of terms in the last two questions, while in Fischbein's work this kind of error is less remarkable in one of the items violating the primitive model. These differences between our study and Fischbein's one might be due to the time passed between the two studies, but also to the fact that our sample is larger and more stratified in terms of school's background. The experimental plan and the methodology implemented in our research give the possibility to discuss past results on which our research field is grounded and to further deepen research on primitive models of multiplication and division nowadays.

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