Capturing flexibility in mental calculation
Design of an instrument

Timo Flückiger

University of Kassel, Germany

Email: flueckiger@uni-kassel.de

Elisabeth Rathgeb-Schnierer

University of Kassel, Germany

Email: rathgeb-schnierer@mathematik.uni-kassel.de

Theme: *Number flexibility*

Keywords: *flexibility in mental calculation, addition and subtraction, elementary arithmetic, standardized interview*

Introduction

Flexibility in mental calculation is a central goal of elementary school mathematics which is also reflected in the large amount of research in this field. In this context, there was an increasing research interest in children's approaches in the last two decades (e.g. Heirdsfield & Cooper, 2004; Rathgeb-Schnierer & Rechtsteiner, 2018; Threlfall, 2009). Various empirical findings show that primary students rarely solve problems flexibly and adaptively; especially after learning standard written algorithm (Selter, 2001; Torbeyns & Verschaffel, 2016). Many students solve problems such as 701 – 698 with a standard written algorithm, even if it is associated with a high number of solution steps and a high error rate (Selter, 2001). Accordingly, students are often not capable to refer to number patterns and relationships as well as specific problem features for solving one or multi-digit problems flexibly (Selter, 2001; Trobeyns et al., 2009). Furthermore, research results suggest that students' creative and flexible use and combination of different strategic means (Rathgeb-Schnierer & Green, 2013) got lost after the introduction of the respective strategies as a sample solution in the classroom (Benz, 2005). Thus, specific approaches for learning arithmetic seem to have a negative impact on students abilities in flexible mental calculation. On the other hand, various studies show that students flexible mental calculation can be supported by appropriate instructional approaches (Grüßing et al., 2013; Nemeth et al., 2021; Rathgeb-Schnierer, 2006; Rechtsteiner-Merz, 2013; Rechtsteiner & Rathgeb-Schnierer, 2017). For the development of perceiving and using number patterns and relationships as well as flexible mental calculation Schütte (2004) emphasizes the importance of the approach "Zahlenblickschulung" (p. 142).[1]

Thus, teaching approaches have a crucial impact on students' abilities in flexible and adaptive arithmetic. Furthermore, the correctness of a problem solution is mediated by a more adaptive strategy use (Nemeth et al., 2021). Although there is broad agreement on the need to support the ability to solve problems flexibly and adaptively, these terms are not consistently defined and operationalized in different studies. Rechtsteiner-Merz (2013) has analyzed the existing approaches (see also Nunes et al., 2016). In this study, we refer to the approach which connects

[1] For further information on this concept see for example Rechtsteiner and Rathgeb-Schnierer (2017). Comparable approaches can also be found in Threlfall (2002).

the notion of flexibility to cognitive elements that sustain the solution process (e.g. Rathgeb-Schnierer & Green, 2013; 2015). Such cognitive elements can be “learned procedures (such as computing algorithms) or recognition of number characteristics (such as number patterns and relations)” (Rathgeb-Schnierer & Green 2019, p. 5). In this vein, we define flexible mental calculation according to Rathgeb-Schnierer and Green (2013): "Only if the tools of solution are linked in a dynamic way to problem characteristics, number patterns, and relationships would we consider as evidence of flexibility in mental calculation" (p. 357).

Aim of the study

There are various research project which investigate flexibility in mental arithmetic (calculation) the cognitive elements that sustain the solution process (Green & Rathgeb-Schnierer, 2020; Rathgeb-Schnierer, 2006; 2010; Rathgeb-Schnierer & Green, 2017; Rechtsteiner-Merz, 2013; Rechtsteiner & Rathgeb-Schnierer, 2017). However, conclusions about these cognitive elements cannot be drawn directly on the basis of manifest characteristics, such as the obtained solution or the described way of solution. Valid conclusions can only be derived from students’ explanations and justifications. So far, no standardized instrument exists for capturing the cognitive elements objectively and reliably. Furthermore, the existing instruments are not sufficiently validated. According to this, already published research results only allow limited generalizable conclusions.

The present study pursues innovative approaches on several levels. It targets to develop and evaluate a standardized, semi-structured interview for second and third graders which allows to capture flexible mental calculation abilities by revealing the cognitive elements that sustain the solution process. Therefore, a semi-structured guideline will be developed which comply the quality criteria of objectivity, reliability and validity. The interview contains activities of sorting problems, reasoning about sorting decisions, comparing and solving problems (Rathgeb-Schnierer, 2006; Rathgeb-Schnierer & Green, 2013; Rechtsteiner & Rathgeb-Schnierer, 2017) For evaluating the interview, it we will conducted and videotaped. The sample comprises students from end of second grade and first half of third grade. Additionally, various other constructs will be surveyed, such as linguistic abilities. With our project we aim to enhance existing methods to capture flexibility in mental calculation by adding an interview instrument which provides a different perspective and allows to be applied to a large sample. With achieving this goal, new options will arise regarding quantitative and qualitative analyses as well as triangulation analysis.

References

Benz, C. (2005). *Erfolgsquoten, Rechenmethoden, Lösungswege und Fehler von Schülerinnen und Schülern bei Aufgaben zur Addition und Subtraktion im Zahlenraum bis 100*. Zugl.: Heidelberg, Pädag. Hochsch., Diss., 2005. *Texte zur mathematischen Forschung und Lehre: Vol. 40*. Franzbecker. http://deposit.dnb.de/cgi-bin/dokserv?id=2713080&prov=M&dok_var=1&dok_ext=htm

Green, M., & Rathgeb-Schnierer, E. (2020). Summing Up: Cognitive Flexibility and Mental Arithmetic. Journal of Mathematics Education, 13(1), 1–17.

Grüßing, M., Schwabe, J., Heinze, A., & Lipowsky, F. (2013). The Effects of Two Instructional Approaches on 3rd-graders' Adaptive Strategy Use for Multi-digit Addition and Subtraction. In A. M. Lindmeier & A. Heinze (Eds.), *Mathematics learning across the life span: Proceedings of the 37th conference of the International Group for the Psychology of Mathematics Education; PME 37; Kiel, Germany, July 28 - August 02, 2013* (pp. 393–401). IPN Leibniz Inst. for Science and Mathematics Education.

Heirdsfield, A. M., & Cooper, T. J. (2004). Factors affecting the process of proficient mental addition and subtraction: case studies of flexible and inflexible computers. *The Journal of Mathematical Behavior*, *23*(4), 443–463. https://doi.org/10.1016/j.jmathb.2004.09.005

Nemeth, L., Werker, K., Arend, J., & Lipowsky, F. (2021). Fostering the acquisition of subtraction strategies with interleaved practice: An intervention study with German third graders. *Learning and Instruction*, *71*, 101354. https://doi.org/10.1016/j.learninstruc.2020.101354

Nunes, T., Dorneles, B. V., Lin, P.-J., & Rathgeb-Schnierer, E. (2016). *Teaching and Learning About Whole Numbers in Primary School. ICME-13 Topical Surveys*. Springer International Publishing. https://doi.org/10.1007/978-3-319-45113-8

Rathgeb-Schnierer, E. (2006). *Kinder auf dem Weg zum flexiblen Rechnen: Eine Untersuchung zur Entwicklung von Rechenwegen bei Grundschulkindern auf der Grundlage offener Lernangebote und eigenständiger Lösungsansätze*. *Texte zur mathematischen Forschung und Lehre: Vol. 46*. Verlag Franzbecker.

Rathgeb-Schnierer, E. (2010). Entwicklung flexibler Rechenkompetenzen bei Grundschulkindern des 2. Schuljahrs. *Journal Für Mathematik-Didaktik*, *31*(2), 257–283. https://doi.org/10.1007/s13138-010-0014-y

Rathgeb-Schnierer, E., & Green, M. (2013). Flexibility In Mental Calculation In Elementary Students From Different Math Classes. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *CERME 8: Proceedings of the Eight Congress of the European Society for Research in Mathematics Education: Manavgat-Side, 2013* (pp. 353–362). Middle East Technical University.

Rathgeb-Schnierer, E., & Green, M. (2017). Profiles of Cognitive Flexibility in Arithmetic Reasoning: A Cross-Country Comparison of German and American Elementary Students. *Journal of Mathematics Education*, *10*(1), 1–16.

Rathgeb-Schnierer, E., & Green, M. (2019). Developing Flexibility in Mental Calculation. *Educação & Realidade*, *44*(2), Article e87078, 1–17. https://doi.org/10.1590/2175-623687078

Rathgeb-Schnierer, E., & Rechtsteiner, C. (2018). *Rechnen lernen und Flexibilität entwickeln: Grundlagen - Förderung - Beispiele*. *Mathematik Primarstufe und Sekundarstufe I + II*. Springer Spektrum. http://dx.doi.org/10.1007/978-3-662-57477-5 https://doi.org/10.1007/978-3-662-57477-5

Rechtsteiner, C., & Rathgeb-Schnierer, E. (2017). "Zahlenblickschulung" as Approach to Develop Flexibility in Mental Calculation in all Students. *Journal of Mathematics Education*, *10*(1). https://doi.org/10.26711/007577152790001

Rechtsteiner-Merz, C. (2013). *Flexibles Rechnen und Zahlenblickschulung: Entwicklung und Förderung von Rechenkompetenzen bei Erstklässlern, die Schwierigkeiten beim Rechnenlernen zeigen*. *Empirische Studien zur Didaktik der Mathematik: Vol. 19*. Waxmann. http://www.content-select.com/index.php?id=bib_view&ean=9783830980377

Schütte, S. (2004). Rechenwegnotation und Zahlenblick als Vehikel des Aufbaus flexibler Rechenkompetenzen. *Journal Für Mathematik-Didaktik*, *25*(2), 130–148. https://doi.org/10.1007/BF03338998

Selter, C. (2001). Addition and Subtraction of Three-digit Numbers: German Elementary Children's Success, Methods and Strategies. *Educational Studies in Mathematics*, *47*(2), 145–173.

Threlfall, J. (2002). Flexible Mental Calculation. *Educational Studies in Mathematics*, *50*(1), 29–47. https://doi.org/10.1023/A:1020572803437

Threlfall, J. (2009). Strategies and flexibility in mental calculation. *ZDM*, *41*(5), 541–555. https://doi.org/10.1007/s11858-009-0195-3

Torbeyns, J., Smedt, B. de, Ghesquière, P., & Verschaffel, L. (2009). Acquisition and Use of Shortcut Strategies by Traditionally Schooled Children. *Educational Studies in Mathematics*, *71*(1), 1–17. http://www.jstor.org/stable/40284582

Torbeyns, J., & Verschaffel, L. (2016). Mental computation or standard algorithm? Children's strategy choices on multi-digit subtractions. *European Journal of Psychology of Education*, *31*(2), 99–116. https://doi.org/10.1007/s10212-015-0255-8