Trends and differences in students' flexibility in arithmetic

Lóa Björk Jóelsdóttir

Aarhus University, Denmark; loaj@econ.au.dk / loho@via.dk

Theme: Flexibility

Keywords: Flexibility, Potential Flexibility, Practical Flexibility, Flexibility Assessment

Introduction and theoretical framework

Researchers argue that solving math tasks efficiently, creatively, with flexibility and with a meaningful choice of strategies has outcompeted mastery of standard algorithms as the ideal strategy. Although, it still requires further research attention, in particular for lower achieving children (Torbeyns & Verschaffel 2016).

The common understanding and focus in different definitions and operationalizing of strategic flexibility and adaptivity is on the knowledge of different solution strategies and the ability to adapt these strategies effectively when solving a problem. This is based on conceptual knowledge of numbers, number relations, and operations (Hickendorff, 2018). Both flexibility and adaptivity are considered important for mathematical proficiency (Hickendorff, 2018; Xu et al., 2017)

The innovative strategies for a given problem are the strategies for that problem that have the fewest steps and with the most simplified computations (Xu et al, 2017, Star & Rittle-Johnson, 2008). These strategies are based on conceptual understanding that according to Hatano and Inagaki (1986) characterise the adaptive experts in contrast to the routine experts characterised by solving problems with speed, accuracy and automaticity.

Previous studies have revealed that different instructional settings influence the students' strategy choice. Students' that were taught in traditional skills-oriented classes, focusing on a specific strategy preferred the school taught strategy (Torbeyns et al, 2009). Even though a student is able to demonstrate an innovative strategy, the student might not use these innovative strategies for solving a given problem (Xu et al, 2017). Thus, Xu et al. distinguish between students' potential flexibility (i.e. knowledge of multiple standard/innovative strategies for solving mathematics problems) and practical flexibility (i.e. ability to implement innovative strategies).

In this study, I will investigate trends and differences in students' potential and practical flexibility with focus on arithmetic, across grade levels and mathematics classes. The purpose is to provide better understanding of the students' flexibility in strategy choice, comparing different age groups and classes, and to assess the usability of a written flexibility assessment adapted from Xu et al. (2017).

Methods

Approx. 2300 students from Grade 3 (765 students, 37 classes), Grade 6 (740 students, 39 classes) and Grade 8 (820 students, 45 classes) in 19 schools in Jutland, Denmark solved the written Flexibility assessment.

The measurement tool is adapted from Tri-phase Flexibility Assessment (Xu et al. 2017), which is developed to measure the students' flexibility solving equations. The tool is developed to measure strategy flexibility with opportunity to distinguish between what students know about different strategies (potential flexibility) and their actual strategy choice (practical flexibility). Xu et al. (2017) pointed towards adaptation to other mathematical domains. In this study, the technique is adapted to arithmetic within the operations addition, subtraction and multiplication.

The measurement consists of three phases aimed at measuring different aspects of flexibility. Practical flexibility is measured in phase one. Here, the students were asked to solve the problem with the strategy they prefer for each item. If solving in their head, they were asked to write down, step by step how they solved the problem. If solving with paper and pencil, the students were asked to include every step in their written notes. Phase two and three provide information on potential flexibility. In phase two, the students were asked to solve the same problems again using another strategy, one or more. In phase three, the students were asked to select which of their strategies from phase one and two they find to be the best/smartest strategy for each item. The instruction in phase three included an explanation of the concept innovative strategy.

The assessment for Grade 3 included eight two- and three-digit addition and subtraction problems (four of each). Grade 6 included three-digit addition problems, two and three digits subtraction problem and one- and two-digit multiplication problems (three of each) and Grade 8 included the same problems as Grade 6, but one more of each operation, total twelve problems.

Students' answers from phase one and two were coded for strategy type: innovative strategy, standard algorithm or other. Further phase two were coded for how many new strategies (0-2 for Grade 3 and 0-4 in Grade 6 and 8). Phase three were coded for if the students selected and innovative strategy or not. All answers were coded by, at least two raters and checked for disagreements.

The student's practical flexibility is calculated by phase one, one point were given for innovative strategy for each item. Max 12 points in Grade 8, 9 points in Grade 6 and 8 points in Grade 3. Potential flexibility is the composite score indicating whether students demonstrated multiple strategies in phase two and were able to identify the innovative one among the strategies from phase one and two.

The project includes surveys for the parents and mathematics teachers of the participating students on attitude towards the use of different strategies in arithmetic.

This study is the first of three sub studies in the project Flexibility in Mathematics with focus on arithmetic. The results about trends and differences between age levels and classes will be studied further in study three. It is also my expectation that the study will contribute with an assessment tool to assess potential and practical flexibility in the domain of arithmetic. The tool is expected to be used in further research of flexibility and as an didacticalal tool for mathematics teachers to assess the students' potential and practical flexibility.

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