The difficulty to see the parts within the whole
(without losing the whole out of sight)

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Abstract applies to the following main themes: number sense (first choice), flexibility

Key words: Part-whole concept, computing without counting, educational design research

General description on research questions, objectives, and theoretical framework

There seems to be consensus that, to quote Resnick (1983, p. 114), conceiving “numbers as compositions of other numbers” forms an important step in early arithmetic thinking. Numerical part-whole thinking allows understanding addition and subtraction as just different ways to deal with the relation of the numbers of a part-whole triple. On that basis, a child may deduce from any known triple at least the thereto directly related addition and subtraction tasks without referring to counting strategies (e.g. 8–5, 8–3, 3+5, 5+3 from knowing 8 as composed of 3 and 5; Gaidoschik, 2019a).

However, for a child whose main approach to numbers so far has been using them to *count*, numbers might well seem to be “tags” (Brissiaud, 1992, p. 65) that adhere to single objects. E.g., the number nine, in this understanding, *does not contain* seven as a part. The child has yet to learn “to disembed seven from nine and treat it as a number separate from nine while leaving it in nine“ (Steffe, 1992, p. 96). Gerster and Schultz (2000, p. 65) stress that the mental *construction* of numerical part-whole thinking is not simply an “empirical abstraction” from working with structured material. We certainly cannot *force* a child to *reflect* numbers in this way. However, it is plausible that the way we teach arithmetic plays a major role in the development of number thinking. In this respect, it might even be "detrimental in the long term" (Brissiaud, 1992, p. 65) if children initially solve addition and subtraction problems by using a counting strategy. In doing so, rather than reflecting on part-whole relations, they tend to concentrate on the single steps of a *process* whose laboriousness might even hinder them to get through to the *“procept”* (Gray & Tall, 1994, p. 129) that includes understanding numbers as composed by numbers.

Against this backdrop, Gaidoschik (2019b) proposed the sketch of a teaching design with a clear focus on elaborating and consolidating part-whole thinking of numbers at least up to 10 in the first weeks of formal instruction. Only in the next step, the design challenges children to solve addition and subtraction tasks, without referring to counting strategies from the very beginning. In the meantime, this design has been put into practice on very small scale. At the TWA-meeting in Leeds, some qualitative results of the evaluation of the first cycle (a second was prevented by the lockdown due to the Covid pandemic) shall be presented, with a focus on the following research questions:

- How do children, especially those with poor number knowledge and competencies at the beginning, respond to teaching that fosters part-whole thinking as a prerequisite for addition and subtraction without counting, as delineated in more detail in Gaidoschik (2009b)?
- Which obstacles in the development of numerical part-whole thinking can be identified in such a learning environment?
- Which conclusions can be drawn for potential improvements of the design?

Methodology

The author realized the teaching experiment in a first grade class of 18 children in a small village in South Tyrol (Italy). The teacher, who had volunteered for this project, had three half-day meetings with the author before the start of the school year to clarify and discuss the main ideas of the design and prepare its implementation. Starting from the second school week, the author visited the class every two weeks to a) observe one math lesson b) interview selected children individually outside the classroom and c) discuss next steps with the teacher. All interviews were videotaped. Due to restricted time capacities, this closely meshed cooperation had to be stopped after six visits. From then on, the teacher worked on her own, yet informing and consulting the author once a month. 2 weeks before the end of year 1, the author interviewed 15 of the 18 children who got parents' consent, for a last time to establish their calculation strategies used for 22 addition and subtraction tasks up to 20.

Results, implications, and relations to number sense

The overall results of the teaching experiment are encouraging. At the end of year one, only one child referred to a counting strategy to solve a basic task up to 10 (in three cases of counting on from the larger). Even the three children who, at the start of the school year, showed the lowest level of arithmetic performance, by the end of grade one had fact retrieval as their main strategy with tasks up to 10. As a backup-strategy, these children typically used their *fingers without counting* them – a strategy that indicates part-whole thinking, albeit still bound to a concrete representation as hypothesized as an intermediate stage by Gerster and Schultz (2000, p. 66).

The microgenetic approach of the first three months allowed for better understanding single children's development, starting from not having yet acquired Cardinality and Order-irrelevance principles (Gelman & Gallistel, 1978) at the start of the school year to – after several weeks of constant targeted classroom work – first indications of part-whole thinking of numbers. These were typically bound to finger representations preferably used in the classroom. For instance, Ada, a girl with very scarce number knowledge at the beginning, after 8 weeks finally was secure to show all numbers up to ten with her fingers without counting them and to verbalize, e.g., eight as being composed of five and three. On that basis, she was able to solve 8–5 without counting, even without using the fingers, by imagining "taking away the full hand". However, she had no clue how to solve, e.g., 8–7. Having no problem to state that seven plus one equals eight, she would not realize seven as part of eight. The interviewer, for his part, at that time failed in his effort to help her "seeing" seven fingers as part of eight fingers. It seemed that she needed the parts represented by visibly *separated* sub-units (embodied in two separate hands) to grasp the parts and the whole at the same time.

As indicated above, she and other children had overcome that special problem at the end of the school year. Still, the observations made during the first cycle seem to implicate that in a second cycle we should put even more emphasis in classroom practice on eliciting early part-whole interpretations beyond the salient "power of five", an interpretation favored by finger representations as well as ten-frames. We still tend to think that it is important for children to have these salient compositions as a starting point and that, for this purpose, finger representations are useful. However, some children seem to need more active support to overcome the restrictions of this "material" when it comes to further *expanding* part-whole thinking, which is at the heart of what may be called "number sense".

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