Developing networks of number relations as a foundation for flexibility with numbers and operations

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1st - number flexibility
2nd - number sense

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General description on research questions, objectives and theoretical framework

This paper is part of a project about the development of flexible calculation in primary school. Its main goal is to contribute to the understanding of how the development of additive calculation flexibility is related to the evolution of students' construction of number concept, based on theoretical frameworks of Threlfall (2009) and Sfard (1991). We intend to answer to the following research question: How is the additive calculation flexibility interconnected with the evolution of students' construction of number concept?

For us, the idea of flexible calculation is associated with mental calculation and problem solving, as to solve a problem in a flexible way implies to notice the numbers involved, how they can relate, and to mobilize strategies that take advantage of the characteristics observed in the numbers (Threlfall, 2009). So, calculation flexibility relates to the discovery of patterns and relationships as students develop the number sense, building a network of numerical relationship (Baroody & Rosu, 2006). Some authors (Heinze et al., 2009; Heirdsfield & Cooper, 2004; Star & Newton, 2009) consider flexible calculation as the choice of the most efficient strategy for a given problem. But, the most appropriate strategy depends on the context but also on the individual characteristics of the students. Although they are in a similar perspective to Threlfall (2009), these authors are distinguished by the fact that they assume that the strategy is selected while Threlfall (2009) states that the strategy emerges, not being the object of choice – named the 'zeroing-in' perspective. This author also warns that the students may not have "the number knowledge they need to sustain the steps of all the emergent calculation-strategies that arise from their noticing and exploratory partial-calculations" (p. 551). He goes further and states that "uncertainty arises from inadequate conceptually-based number knowledge" (p. 551).

Conceptually a number may be structurally conceived as an object or operationally as a process and these approaches are complementary. Thinking about numbers, Sfard (1991) states that "the ability of seeing […] a number both as a process and as an object is indispensable for a deep understanding" (p. 5). According to this author three stages can be distinguished in concept development: interiorization, condensation and reification.

Methodology

This study follows a qualitative approach within an interpretive paradigm. Its methodology of *design research* is part of a perspective of learning design, in order to produce local theories of teaching and learning sequences that are resources and references available to inform the practices of teachers and researchers (Gravemeijer, 2015).

The data were collected in a class with 26 students, of a public primary school in Lisboa, in its first and second grade. The Project team defined a sequence of tasks with the aim to develop the calculation flexibility in addition and subtraction problems. The process of tasks elaboration included previous testing of some (namely the ones focused in this paper), through clinical interviews with students of the same grade. The data collection was made through participant observation of the authors of this paper, which drew up field notes supported by video recording, subsequently transcribed. The written records of the students were also collected. All these data were analyzed and triangulated. By ethical reasons, the students' names were changed to ensure confidentiality.

This paper is focused on two tasks, one explored in the 1st grade and another in the 2nd grade in the same class, with the same teacher. In 1st grade, the task consists in challenging the students to present different expressions representative of the day number (19), which was a routine activity explored and discussed in a collective moment, at the beginning of the day. In 2nd grade, the task consists in separating different cards with expressions of sums and differences into two groups, one with the known cards and another one with the unknown expressions, in order to establish number relations to deduce unknown values. In this class, the students began by exploring the task autonomously, followed by a moment of collective discussion.

In the data analysis we try to understand how the students approach each of the situations in order to solve the problem and how this is connected with their conceptual development and the additive calculation flexibility.

Outcomes/results and implications based on the research results

The data show an evolution in the students' flexibility of calculation interconnected with their conceptual development. Students establish numerical relationships, namely, in the 1st grade, the double and the transformation of the expressions by using several operations (for example, 5+5+5+4 and 3x5+4), and in the 2nd grade, the relationships between different decompositions of the numbers that are used to deduce unknown values. We conclude, therefore, that the flexibility of calculation is substantively supported by the creation and expansion of networks of number relations (Gravemeijer et al., 2016) performed in the condensation and reification stages present in the model of Sfard (1991).

Relation to the theme chosen

This paper focuses mainly the flexibility arguing that its foundation is based on creation and expansion of networks of number relations, strictly related with the development of number sense and structure sense. This flexibility occurs in the last stages of number concept development.

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