

Estimation: The missing competence in the mathematics experiences of year-one children

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As part of our work on the Foundational Number Sense (FoNS) project, we undertook the development of a simple to operationalise framework for analysing the number-related learning opportunities received by year-one children (Andrews & Sayers, 2015). The eight categories of FoNS bridge the gap between those number-related understandings innate to all humans and forms of number sense typically associated with functional numeracy. Consequently, each FoNS competence is not only a prerequisite for later mathematical learning but dependent on instruction. One of these categories, estimation, has been identified as one of the three most important mathematical skills (Sriraman & Knott, 2009) and yet, as we discuss below, it is effectively absent from the curricula and textbooks that underpin the teaching and learning of year-one mathematics. Moreover, possibly a consequence of the above, during the early months of the project, a small set of serendipitously available year-one lessons from a number of European countries were analysed against the different FoNS categories and, across all systems, estimation was effectively invisible.

The ability to estimate is widely recognised not only as a core skill of everyday life (White & Szűcs, 2012) but also a key determinant of later arithmetical competence, particularly in respect of novel situations (Booth & Siegler, 2008; Holloway & Ansari, 2009). However, the development of the ability to estimate is not a chance phenomenon but requires intervention (Joram, Gabriele, Bertheau, Gelman & Subrahmanyam, 2005; Peeters, Degrande, Ebersbach, Verschaffel & Luwel, 2016; White & Szűcs, 2012). Unfortunately, the teaching of estimation has been, historically, a neglected skill with textbooks colluding in this omission (Reys, Rybolt, Bestgen & Wyatt, 1982) by offering incomplete or inappropriate models (Joram, Subrahmanyam & Gelman, 1998).

Broadly speaking, estimation takes four forms; computational estimation, number line estimation, quantity estimation and measurement estimation. Of these, number line estimation and quantity estimation are, we argue, lower level skills than computational estimation or measurement estimation and, as such, comprise the estimation elements of FoNS. That said, being able to undertake computational estimation is an essential life skill (Sekeris, Verschaffel & Luwel, 2019) and is, despite teacher scepticism (Alajmi, 2009), an important aid to children's understanding of both place value and standard algorithms (Sowder, 1992). It is a skill that develops with age (Lemaire & Brun, 2014) but is an under-investigated area of arithmetic-related research (Lemaire & Lecacheur, 2011). Number line estimation, which draws on a child's developing ability to exploit reference points (Sullivan & Barner, 2014), is a strong predictor of both mathematical learning difficulties (Siegler & Opfer, 2003) and mathematical achievement (Schneider, Grabner & Paetsch, 2009). Instruction with respect to number line estimation is important if young children's logarithmic estimations of quantity

are to be replaced by linear (Siegler & Opfer, 2003), although others have argued that the logarithmic/linear distinction is less a developmental issue than one related to strategy choice (Ebersbach, Luwel, Frick, Onghena & Verschaffel, 2008). Quantity (or numerosity) estimation is the ability to estimate the number of objects in a set. It is a skill that diminishes in accuracy as the numerosity of the set of objects grows (Smets, Sasanguie, Szűcs & Reynvoet, 2015). The ability to estimate quantities is closely tied to the ability to count (Barth, Starr & Sullivan, 2009) and has a developmental trajectory similar to number line estimation (Sella, Berteletti, Lucangeli & Zorzi, 2015). Interestingly, the evidence indicates that young children tend to exploit linear mappings in continuous conditions and logarithmic in discrete (Odic, Libertus, Feigenson & Halberda, 2013). While measurement estimation is an important life skill, with many users of mathematics using it as an everyday part of their professional decision making based on reference or anchor points (Jones & Taylor, 2009), it remains a neglected research field (Joram, Subrahmanyam & Gelman, 1998). It is known that children who employ references model to their estimates are more accurate than those who do not (Joram, Gabriele, Bertheau, Gelman & Subrahmanyam, 2005) and that context familiarity improves estimates (Jones, Gardner, Taylor, Forrester & Andre, 2012).

In this paper we will examine the materials available to support the teaching of estimation to year-one children. In particular, we will summarise how the curricula of ten European countries present estimation alongside analyses of six textbooks currently used in the year-one classrooms of England and Sweden. Two of these are authored by nationals of the two countries, while the remaining four are adaptations of textbooks drawn from countries typically seen as successful on international tests of achievement. The analyses indicate, confirming research undertaken nearly forty years ago, that across the board, estimation in any form is absent from year-one children's opportunities to learn.

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