Equivalence of fractions: kinds of students' reasoning

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Research questions and framework

In our paper, we present a research study about student reasoning related to equivalence of fractions at primary level (grade 5). What kind of reasoning do students employ to check that two fractions are equivalent or not? What mathematical knowledge do such arguments require? What are the limits or potentialities of this mathematical knowledge?

In the French curriculum, fractions are studied as "splitting the unit" meaning in primary school. This meaning is a limited meaning of part-whole meaning linked to measure (Behr & al., 1983): the whole is systematically a "visible" unit related to areas (with circular or rectangular models) or related to lengths (with lines or number-line). Nonetheless it allows comparison and operations with fractions that rely on multiplicative relations between values of parts of the unit or numbers of parts of the unit. The nature of such multiplicative relations can be examined regarding mathematical knowledge about multiplicative structures (Van de Walle & al., 2010; Greer, 1992, Greer, 1988), and product of measure (Behr & al., 1994; Chambris, 2019; Chambris & al., 2019).

Methodology

In collaboration with a teacher, we designed and experimented different mathematical tasks around comparison of fractions in a grade 5 class (10-11 years old). It is important to note that the students of this class were quite used to use to using area models (circular, rectangular models) to represent fractions, including improper fractions. The sessions (4 sessions of about one hour each) were filmed and transcribed – written works of students were collected. We analyse transcriptions and written works in order to identify diverse strategies and arguments students produced to compare fractions within the different mathematical tasks. In our presentation we focus on the equivalence of two fractions (1st session of the sequence).

Results: Two kinds of reasoning about equivalence of fractions

In the context of the tasks given about equivalence of two fractions whose one denominator is a multiple of the other one, we identified two kinds of reasoning from students.

A large majority of students seem to proceed by identifying a multiplicative relation between denominators and interpret it as "equal sized groups of lowest subdivisions *included* in largest subdivisions". In order to compare fractions whose denominators $m = p\ n$ and n, students first establish that "$\frac{1}{n}$" includes *p times* "$\frac{1}{m}$". This multiplicative inclusion relation relies on "splitting the unit" meaning students manage to extend to "splitting a subdivision of the unit". Then such reasoning relies on a number of equal-sized groups of p lowest subdivisions of the unit "included" in each of n largest subdivisions of the unit. That kind of reasoning relies on *multiplicative inclusion* *relations* and remains close to counting and repeated addition: it allows counting groups of lowest subdivisions. All students but one resorted to this kind of reasoning.

The outlier student employed another strategy based on the identification of a *multiplicative comparison relation* between largest and lowest subdivisions of the unit. We can formulate it as follow: "$\frac{1}{m}$" is *p times smaller* than "$\frac{1}{n}$" - so it is needed *p times as many* "$\frac{1}{m}$" to obtain the same value as in " $\frac{1}{n}$". Such reasoning is different from that described above, especially in terms of multiplicative comparison relationship between fractions involved in this task.

The difference between these two kinds of reasoning manifests itself in the difficulties encountered by teacher to spread such a rationale in the classroom. In this episode, tension exists between the discourse of the teacher who try to formulate multiplicative comparison relations "*p times smaller than*" "*p times as many*" and students who tries to reformulate multiplicative relations in a sense of inclusion: "*p times ... in*", and counting subdivisions or groups of subdivisions. This episode highlights differences between multiplicative inclusion relations (*n* subdivisions of a unit included in another subdivision of the same unit) and multiplicative comparison relations (a subdivision of a unit *n* times smaller than or *n* times as large as another subdivision of the same unit).

Discussion and perspectives

We relate such differences of reasoning about equivalence of fractions to the distinction between multiplicative inclusion related to equal-sized groups and multiplicative comparison (Van de Walle & al., 2010; Greer, 1988; Greer, 1992). Multiplicative inclusion relations seem to be more easily operated by students: it appears in continuity with previous knowledge about "splitting the unit" meaning of fractions. Yet we hypothesize that multiplicative comparison relations are necessary to conceptualize in order to access to the sense of fractions as numbers. Accounting for account a reconceptualization of multiplication as a product of measure (Behr & al. 1994, Chambris 2019, Chambris & al. 2019), we try and see how such multiplicative comparison relation could be taught and learned by students (with whole numbers and fractions). The commutative property of the multiplication expressed as "taking *n* times more/less than a value *b* which is *n* times smaller/larger than a given value *a* gives something which is equal to *a"* (which is below the second kind of reasoning about equivalence of fractions) could be an interesting point to investigate. Nonetheless, does "splitting the unit" meaning make it easy to access such a rationale? We think that better ways could exist: maybe a *commensuration* meaning of fraction (see our second proposal of paper)?

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